Fundamentals of Numerical Analysis

Course description:

This is the first course in numerical analysis for graduate students. The main objectives of this course include:

(1) Development and applications of numerical methods when analytical techniques are not available;
(2) Development of a conceptual framework for analysis of methods to fix the problem;
(3) Discrete calculus and approximations;
(4) Tradeoffs between accuracy and computational cost;

Course outline:

1. Interpolation (3 hrs)
   (1) Lagrange Polynomials
   (2) Polynomial interpolations; Splines
2. Numerical Differentiation (4 hrs)
   (1) Construction of finite difference scheme, order of accuracy
   (2) Modified wavenumber as a measure of accuracy
   (3) Pade approximation
   (4) Matrix representation of finite difference schemes
3. Numerical Integration (8 hrs)
   (1) Trapezoidal rule; Simpson's rule; error analysis and mid-point rule
   (2) Romberg integration and Richardson's extrapolation
   (3) Adaptive quadrature; Gauss quadrature
4. Numerical Solution of Ordinary Differential Equations (10 hrs)
   (1) Initial value problems; numerical stability analysis, model equation
   (2) Accuracy; phase and amplitude errors
   (3) Runge-Kutta type formulas, multi-step methods; implicit methods
   (4) System of differential equations; stiffness
   (5) Linearization for implicit solution of non-linear differential equations
   (6) Boundary value problems, shooting, direct methods, non-uniform grids, eigenvalue problems
5. Partial Differential Equations (10 hrs)
   (1) Finite-difference solution of partial differential equations
   (2) Modified wavenumber and Von Neumann stability analysis, modified equations analysis
   (3) Alternating direction implicit methods; non-linear equations; iterative methods for elliptic PDE's

6. Fourier Series (6 hrs)
   (1) Trigonometric Interpolation;
   (2) FFT Algorithm;
   (3) Applications of DFT;

Textbook:

Parviz Moin “Fundamentals of Engineering Numerical Analysis”, Cambridge University press

Michael T. Heath “Scientific Computing: An Introductory Survey”, McGraw Hill

Grading:

Homeworks (55%); Midterm exam (%15); Final exam (%30)