

崔茂培：幾何分析專題

Nature of the course 加開選修		Area： 分析，幾何與拓樸			
Course number		Section number	免填	Number of credits	3
Course title	課程名稱：幾何分析專題 Topics in Geometric Analysis				
Instructor	教授：Mao-Pei Tsui				
開設學期： 全學年	上課時間： 星期 節次		開課對象： 皆可		

I. ***Contents** : Geometric evolution equations have seen tremendous progress in the last 30 years. In 1964 the seminar paper of Eells and Sampson introduced the harmonic heat flow and used it to prove the existence of harmonic maps into targets with non-positive sectional curvature. The year 1982 marked the beginning of Ricci flow with the appearance of Hamilton's paper on 3-manifolds with positive Ricci curvature. In 1984, Gerhard Huisken studied the mean curvature flow of convex hypersurfaces. The main topics of this course are the interaction between nonlinear partial differential equations and differential geometry, especially in areas where the deformation of geometric objects by nonlinear parabolic equations can bring a new understanding to the geometric problems. Such connections show up in Hamilton-Perelman's proof of Poincare and Thurston's geometrization conjecture via Ricci flow; as well as in Huisken and Ilmanen's proof of the Riemannian Penrose conjecture via inverse mean curvature flow.

We will cover the following topics.

- (1) The basic backgrounds and techniques to study geometric evolution equations
- (2) Go over the fundamental papers in harmonic heat flow, Ricci flow and mean curvature flow
- (3) Understanding of the long time existence and singularity analysis of various geometric evolution equations

II. **Course prerequisite** :

III. ***Reference material (textbook(s)) :**

Book: Hamilton's Ricci Flow by Bennett Chow, Peng Lu , and Lei Ni

Papers:

1. Huisken, Gerhard. Flow by mean curvature of convex surfaces into spheres. Journal of Differential Geometry 20 (1984), no. 1, 237--266
2. Hamilton, Richard S. Three-manifolds with positive Ricci curvature. Journal of Differential Geometry 17 (1982), no. 2, 255--306.

3. Additional references will be provided as we go along.

IV. ***Grading scheme :**

作業50% 報告50%

V. ***Course Goal :**

The aim of this course is to provide students with an indepth introduction to a number of important aspects of geometric evolution equations. On successful completion of this course the students should be able to:

- Compute the evolution of geometric quantities under various geometric evolution equations
- Understand the parabolic maximum principle for scalar function and tensors
- Understand the proof of the long time existence of various geometric evolution equations
- Understand the singularity analysis of various geometric evolution equations