## Units Units Units Units


table 9.1
Fig. 2.: Atomic Units and Their SI Equivalents with a) term,
table 10.2
Results of Various Calculations of the Ground-State Electronic Energy of $\mathrm{H}_{2}^{+\mathrm{a}}$

| $\phi$ | $E_{\min } / E_{\mathrm{h}}$ | $R_{\mathrm{eq}} / a_{0}$ |
| :--- | :--- | :--- |
| $1 s(\zeta=1.000)$ | -0.56483 | 2.49 |
| $1 s(\zeta=1.238)$ | -0.58651 | 2.00 |
| $1 s(\zeta=1.000)+a 2 p_{z}(\zeta=1.000)$ | -0.56591 | 2.00 |
| $1 s(\zeta=1.247)+b 2 p_{z}(\zeta=1.247)$ | -0.59907 | 2.00 |
| $1 s(\zeta=1.2458)+c 2 p_{z}(\zeta=1.4224)$ | -0.60036 | 2.00 |
| $1 s(\zeta=1.244)+c_{1} 2 p_{z}(\zeta=1.152)+c_{2} 3 d_{z^{2}}(\zeta=1.333)^{\mathrm{b}}$ | -0.6020 | 2.00 |
| Exact $^{\mathrm{c}}$ | -0.60264 | 2.00 |

a the table. rk, 1977.
Molecular Ion.

| Property | Atomic unit | SI equivalent | 1 hartree |
| :--- | :--- | :--- | :--- |
| Mass | Mass of an electron, $m_{\mathrm{e}}$ | $9.1094 \times 10^{-31} \mathrm{~kg}$ | 27.2114 eV |
| Charge | Charge on a proton, $e$ | $1.6022 \times 10^{-19} \mathrm{C}$ | $627.5095 \mathrm{kcal} / \mathrm{mol}$ |
| Angular momentum | Planck constant divided by $2 \pi, \hbar$ | $1.0546 \times 10^{-34} \mathrm{~J} \cdot \mathrm{~s}$ |  |
| Length | Bohr radius, $a_{0}=\frac{4 \pi \epsilon_{0} \hbar^{2}}{m_{\mathrm{e}} e^{2}}$ | $5.2918 \times 10^{-11} \mathrm{~m}$ | What is energy of |
| Energy | $\frac{m_{\mathrm{e}} e^{4}}{16 \pi^{2} \epsilon_{0}^{2} \hbar^{2}}=\frac{e^{2}}{4 \pi \epsilon_{0} a_{0}}=E_{\mathrm{h}}$ | $4.3597 \times 10^{-18} \mathrm{~J}$ | $\mathrm{H}^{+}$and H? |
| Permittivity | $\kappa_{0}=4 \pi \epsilon_{0}$ | $1.1127 \times 10^{-10} \mathrm{C}^{2} \cdot \mathrm{~J}^{-1} \cdot \mathrm{~m}^{-1}$ |  |

## Questions

- Why do we call adiabatic
- Delocalization and kinetic energy
- Why does using a smaller atomic wavefunction help?
- Why does adding a 2 p orbital help but not 2 S orbital?


## Adiabatic Approximation

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From Wikipedia, the free encyclopedia

An adiabatic process is any process occurring without gain or loss of heat within a system (i.e. during the process the system is thermodynamically isolated- there is no heat transfer with the surroundings). This is the opposite of a diabatic process, where there is heat transfer. A key concept in thermodynamics, many rapid chemical and physical processes are described or approximated in this way. Such processes are usually followed or preceded by events that do involve heat transfer (i.e. are non-adiabatic). Examples include electron-transfer.

## Adiabatic : no heat transfer between electron and nuclei

Heat: kinetic energy!

Kinetic energy of the nuclei does not effect the electron energies

## Questions

- Why do we call adiabatic
- Delocalization and kinetic energy
- Why does using a smaller atomic wavefunction help?
- Why does adding a 2 p orbital help but not 2 S orbital?


## Answers Delocalization

- Now the electron distribution spreads, the position uncertainty increases. According to Uncertainty Principle, the momentum uncertainty will decrease, resulting lower momentum, and lower kinetic energy. (uncertainty and absolute value are the same or not?)
- Delocalization could become covalent bond
- why delocalization lower K.E?I guess the probability curve between two protons become more smooth: then input the new wavefunction :we will get an smaller K.E term because the property of gradient square term in Hamiltonian!(smoother wave lower energy) always work:if the potential term get more important(charge goes up). the effect of delocalization maybe not enough!

$$
T=\frac{p^{2}}{2 m}=\frac{h^{2}}{2 m \lambda^{2}} .
$$

## Using Particle in a box

Consider the one-dimension particle in a box
$V(x)=\{0,0<x<L$ $\infty$. otherwise , : potential energy, $V(x)=0$
.he only need to consider the kinetic elegy

## $\partial(x)=\sqrt{\frac{2}{2}}$ <br> $\sin \left(\frac{n \pi}{2} x\right)$ <br> $E_{n}=\frac{n^{2} h^{2}}{8 m L^{2}}$

Thus, as $\mathcal{L}$ increases $\Rightarrow$ more debcalized. $\Rightarrow$ E decreases $\Rightarrow$ kinetic energy decreases

## Delocalization and Kinetic Energy

$$
\hat{T}=-\frac{\hbar}{2 m} \frac{d^{2}}{d r^{2}}
$$

Kinetic energy is the average of the second derivative
$\hat{P}=i \hbar \frac{d}{d r} \quad \begin{array}{r}\text { Momentum is the average of the derivative } \\ \text { de brogle's relation } p=\frac{h}{\lambda} \text { where } \lambda \text { is wavelength }\end{array}$


Localized


Delocalized

Area have to be kept same so what happens to the wavefunction?

Delocalization and Kinetic Energy 2


## Additional Orbitals

## ABLE 10.2

esults of Various Calculations of the Ground-State Electronic Energy of $\mathrm{H}_{2}^{+}$a

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| Exact $^{\mathrm{c}}$ | -0.60264 | 2.00 |

The molecular orbitals are of the form $\psi_{\mathrm{b}}=c_{\mathrm{A}} \phi_{\mathrm{A}}+c_{\mathrm{B}} \phi_{\mathrm{B}}$, where $\phi$ is given in the table. Mulliken, R. S., Ermler, W. C. Diatomic Molecules. Academic Press: New York, 1977. Bates, D. R., Ledsham, K., Stewart, A. L. Wave Functions of the Hydrogen Molecular Ion. hilos. Trans. Roy. Soc. London, Ser. A. 246, 215 (1953).

## Why Smaller Orbital Helps

- 2.A larger atom has more electrons and orbitals, and more coupling between orbitals, which are too complicated to use the approximation.

2:why a smaller wave function help? the electron get closer to proton:potential decrease ; thought the faster decreasing exponential term means higher K.E. just opposed to the effect of the first question! (how can we do that? what's the meaning of parameter?)

The smaller atomic wavefunction means the higher effective charge. (i.e. the term in wavefunction $\left.e^{-Z_{e f f} r / a_{0}} ; e^{-1 r / a_{0}} \rightarrow e^{-1.238 r / a_{0}}\right)$. The higher effective charge is natural for our physical intuition since that the electron faces two nucleus rather than

## Using smaller Orbitals

ABLE 10.2
desults of Various Calculations of the Ground-State Electronic Energy of $\mathrm{H}_{2}^{+}$a

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| $1 s(\zeta=1.238)$ smaller orbital | -0.58651 | 2.00 |

Growth of density at interatomic regions

Decrease of density from region near nucleus

Bonding Orbital

The bonding orbital has sharper structure near the nuclei

Better described by a smaller coefficient

Look at what happens with smaller orbital



$$
\begin{aligned}
& \text { (1). When } 15(\xi=1,238) \text { and } \operatorname{Ref}\left(a_{0}=2,00, S_{1}=0,23\right) \\
& \text { (2) When } 1 S(\xi=1,000) \text { and } \operatorname{Ref}\left(a_{0}=2,49, S_{2}=0.288\right. \text {. } \\
& \text { For } E_{+}(R)=\frac{H(R)_{A A}+H(R)_{A B}}{1+S(R)} \quad \frac{\frac{4}{E}}{E_{15}} \frac{\phi_{B}}{E_{+}}
\end{aligned}
$$

## Why 2P orbital helps but 2 S does not 1

- 3 :why adding 2 p but 2 s is useful? because different type of orbital can hybrid to lower energy. 1 s and 2 s are orthogonal so adding 2 s is useless! (hybrid is not enough, we need to shirk the orbital using parameter)
1 s and 2 p are also orthogonal
- 3.2p orbital is more "orthogonal" to 1 s orbital than 2 s orbital is to 1 s orbital.


## Why 2P orbital helps but 2 S does not 2

calculation, we never consider the angular contribution. According to quantum mechanics, s-orbit is spherical symmetry, while p-orbit is cylindrical symmetry. From the view of the electron, considering Born-Oppenheimer approximation (i.e. regard the nucleus is moving much slower than electron or say almost fixed at some place), it sees two nucleus lying on an axis; therefore, the potential due to this two nucleus should be cylindrical symmetry. The angular parts do contribute to the calculation! 2 p seems to be more appropriate to describe the $\mathrm{H}_{2}^{+}$system. Thus, only considering spherical symmetry- s-orbit, may not give the correct result even adding the contribution of different principle quantum number. That is the reason why adding $2 p$ can increase more accuracy than 2 s .

## Using $2 \mathrm{p}_{\mathrm{z}}$ Orbitals Hand waving

$1 s(\zeta=1.000)$
$1 s(\zeta=1.238)$
$1 s(\xi$


Bonding Orbital

## Polarization orbital!

| -0.56483 | 2.49 |
| :--- | :--- |
| -0.58651 | 2.00 |
| -0.56591 | 2.00 |

The bonding orbital has asymmetric distribution around nuclei

Better described by a $2 p_{z}$ orbital

## Addition of 2 S Orbitals what will it do?




2 s orbitals are much bigger than the 1 s orbital, but it is still symmetric around the H atom center

## Basis Set

- If you use more atomic orbitals to define the molecular orbital usually the energy gets closer to the exact solution

Using a bigger basis set to describe the system

- However bigger basis set you need more time to calculate.
- Changing coefficient on basis set is also important


## Diatomic Molecules Chemist's view

## Linear $\mathrm{H}_{2}$ <br> antiBonding

Anti Bonding

Fragment 1

Fragment 1


Fragment 2


Fragment 2

Bonding

## $\mathrm{H}_{2}{ }^{+}, \mathrm{H}_{2}, \mathrm{He}_{2}{ }^{+}, \mathrm{He}_{2}$


$\mathrm{H}_{2}{ }^{+} \mathrm{R}_{\mathrm{eq}}=1.06$ Angstrom Binding Energy $61 \mathrm{kcal} / \mathrm{mol}$

$\mathrm{He}_{2}{ }^{+} \mathrm{R}_{\mathrm{eq}}=1.08$ Angstrom Binding Energy $50 \mathrm{kcal} / \mathrm{mol}$

$\mathrm{H}_{2} \mathrm{R}_{\mathrm{eq}}=0.74$ Angstrom
Binding Energy $105 \mathrm{kcal} / \mathrm{mol}$

$\mathrm{He}_{2} \mathrm{R}_{\mathrm{eq}}=$ Infinity not bound Binding Energy 0 Kcal mol

## Polyatomic Molecules

## Linear $\mathrm{H}_{3}$

Fragment 1

Fragment 2

Fragment 1


Bonding

## Triangle $\mathrm{H}_{3}$

Anti Bonding



Fragment 2
Non Boṇiding



## **** <br> 



$\qquad$


## Linear $\mathrm{H}_{3}$ vs Triangular $\mathrm{H}_{3}$

Anti Bonding


## $\mathrm{H}_{4}$ Square


$\bullet$


Fragment 2


Fragment 2

## $\mathrm{H}_{4}$ square



Fragment 1


Fragment 2

## Water $\mathrm{H}_{2} \mathrm{O}$

You must think of symmetry:
Rotation of bisecter axis Mirror at the water plane Mirror at the middle
Hydrogens make linear combination: plus and minus pair like bonding and antibonding orbital

| $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{O} 2 s$ | $\mathrm{O} 2 p_{x}$ | $\mathrm{O} 2 p_{y}$ | $\mathrm{O} 2 p_{z}$ | $\mathrm{H} 1 s_{\mathrm{A}}$ | $\mathrm{H} 1 s_{\mathrm{B}}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $E$ | $\mathrm{O} 2 s_{1}$ | $\mathrm{O} 2 p_{x}$ | $\mathrm{O} 2 p_{y}$ | $\mathrm{O} 2 p_{z}$ | $\mathrm{H} 1 s_{\mathrm{A}}$ | $\mathrm{H} 1 s_{\mathrm{B}}$ |
| $\mathrm{C}_{2}$ | $\mathrm{O} 2 s$ | $-\mathrm{O} 2 p_{x}$ | $-\mathrm{O} 2 p_{y}$ | $\mathrm{O} 2 p_{z}$ | $\mathrm{H} 1 s_{\mathrm{B}}$ | $\mathrm{H} 1 s_{\mathrm{A}}$ |
| $\sigma_{\mathrm{v}}$ | $\mathrm{O} 2 s$ | $\mathrm{O} 2 p_{x}$ | $-\mathrm{O} 2 p_{y}$ | $\mathrm{O} 2 p_{z}$ | $\mathrm{H} 1 s_{\mathrm{B}}$ | $\mathrm{H} 1 s_{\mathrm{A}}$ |
| $\sigma_{\mathrm{v}}^{\prime}$ | $\mathrm{O} 2 s$ | $-\mathrm{O} 2 p_{x}$ | $\mathrm{O} 2 p_{y}$ | $\mathrm{O} 2 p_{z}$ | $\mathrm{H} 1 s_{\mathrm{A}}$ | $\mathrm{H} 1 s_{\mathrm{B}}$ |

## Potential Energy Surface Reaction

## Kaito Takahashi

## Water Molecule

## Potential Energy Curve





## Water Molecule: Stretch Stretch Potential Energy Surface



ROH1


## Water Molecule: Stretch Bend Potential Energy Surface



Vibration

## Potential Energv Curve

## © 140.109.112.238:22 - Tera Term VT

Eile Edit Setup Control Window Resize Help

Scan completed.
Summary of the potential surface
N RH1H2 SCF

| 1 | 0.2500 | -0.37196 |
| ---: | ---: | ---: |
| 2 | 0.3500 | -0.81916 |
| 3 | 0.4500 | -1.00834 |
| 4 | 0.5500 | -1.09016 |
| 5 | 0.6500 | -1.12120 |
| 6 | 0.7500 | -1.12655 |
| 7 | 0.8500 | -1.11854 |
| 8 | 0.9500 | -1.10365 |
| 9 | 1.0500 | -1.08542 |
| 10 | 1.1500 | -1.06576 |
| 11 | 1.2500 | -1.04576 |
| 12 | 1.3500 | -1.02600 |
| 13 | 1.4500 | -1.00682 |
| 14 | 1.5500 | -0.98838 |
| 15 | 1.6500 | -0.97078 |
| 16 | 1.7500 | -0.95407 |
| 17 | 1.8500 | -0.93827 |
| 18 | 1.9500 | -0.92338 |
| 19 | 2.0500 | -0.90939 |
| 20 | 2.1500 | -0.89628 |
| 21 | 2.2500 | -0.88403 |
| ---1 | --------1 | -1 |

## How to quantify Minimum?

- At the minimum the first derivative is zero and the second derivative is always positive

Check the Hessian (second derivative)
In Gaussian you can use the freq keyword

## Harmonic Oscillator

$$
\begin{gathered}
F=m \frac{d^{2} x}{d t^{2}}=-k x \quad k: \text { force constant } \\
\omega=\sqrt{\frac{k}{m}} \quad E_{n}=\left(n+\frac{1}{2}\right) \hbar \omega \quad n=0,1,2, \ldots
\end{gathered}
$$



## Normal Modes of Water

Symmetric stretch


Antisymmetric stretch


Bend

Reaction

# $\mathrm{F}+\mathrm{HCl} \rightarrow \mathrm{FH}+\mathrm{Cl}$ Potential Energy Surface 

 Hydrogen Abstraction Reaction, break a HCl bond form a HF bond


Barrier

$\mathrm{FH}+\mathrm{Cl}$

## Potential Energy Curve along IRC




You need enough energy to go over the potential barrier to react!
The minimum energy path that connect the barrier with reactant and product is called intrinsic reaction path
For $\mathrm{F}+\mathrm{HCl}$ case you need about $5 \mathrm{kcal} / \mathrm{mol}$ of for reaction to occur

## How to quantify Barrier?

- At the barrier the first derivative is zero and the second derivative is negative for one degree of freedom and the rest are always positive. In Gaussian you can use the freq keyword to check if there is imaginary frequency


## Harmonic Oscillator approximation

 at barrier

$$
F=m \frac{d^{2} x}{d t^{2}}=-k x \quad k: \text { force constant } \quad \omega=\sqrt{\frac{-k}{m}}
$$

In the case of barrier the force constant k is negative so omega is imaginary

## Transition State Theory

- Once you have the barrier you can estimate the thermal rate constant of the reaction from the calculation results! (Assuming that the reaction proceeds along the IRC)

$$
A+B \rightarrow(A B)^{+} \rightarrow \text { product }
$$

Activated Complex
(Geomtry at Potential Barrier)

$$
-\left(\frac{d[A]}{d t}\right)=\frac{k_{B} T}{h} K^{+}[A][B]
$$

$K^{+}$is the thermal rate constant that depends on temperature

# Things Needed To Calculate Rate Constant 

$$
K^{+}=\frac{Q\left[(A B)^{+}\right]}{Q[A] Q[B]} \exp \left[-\frac{\Delta E}{k_{B} T}\right]
$$

Q is the partition function of the respective molecules DE is the energy difference from the reactant to the barrier


## Partition Function 1

$$
Q_{\text {trans }}=\left(\frac{2 \pi m k_{B} T}{h^{2}}\right)^{3 / 2} V=\left(\frac{2 \pi m k_{B} T}{h^{2}}\right)^{3 / 2} \frac{k_{B} T}{P}
$$

Usually you consider 1atm pressure

$$
\begin{aligned}
Q_{\text {electronic }} & =D_{0} \exp \left[-\varepsilon_{0} / k_{B} T\right]+D_{1} \exp \left[-\varepsilon_{1} / k_{B} T\right] \\
& +D_{2} \exp \left[-\varepsilon_{2} / k_{B} T\right]+\ldots \\
& =D_{0}
\end{aligned}
$$

D is the degeneracy of the electronic state, usually only consider ground electronic state

## Partition Function 2

$$
Q_{\text {rot }}=\frac{\pi^{1 / 2}}{\sigma_{r}}\left(\frac{T^{3 / 2}}{\left(\theta_{A} \theta_{B} \theta_{C}\right)^{1 / 2}}\right) \quad \theta_{A, B, C}=\frac{h^{2}}{8 \pi^{2} I_{A, B, C} k_{B}}
$$

$I$ is the moment of inertia around the respective axis

$$
Q_{v i b}=\prod_{i}^{N v i b} \frac{1}{1-\exp \left[-\frac{h v_{i}}{k_{B} T}\right]}
$$

$v_{\mathrm{i}}$ is the vibrational frequency of the i-th vibrational mode

## Normal Mode Vibration

First we consider mass - weight Cartesian Coordinates
$q_{1}=m_{1}^{1 / 2} \Delta X_{1} ; q_{2}=m_{1}^{1 / 2} \Delta Y_{1} ; q_{3}=m_{1}^{1 / 2} \Delta Z_{1} ; q_{4}=m_{2}^{1 / 2} \Delta x_{2} ; \ldots q_{3 N}=m_{N}^{1 / 2} \Delta Z_{N}$
Next expand the potential energy to second term near equilibrium

$$
V=V(0)+\sum_{I}^{3 N} K_{I} q_{I}+\frac{1}{2} \sum_{I}^{3 N} \sum_{J}^{3 N} K_{I J} q_{I} q_{J}=\frac{1}{2} \sum_{I}^{3 N} \sum_{J}^{3 N} K_{I J} q_{I} q_{J}
$$

Total energy is given as $E=\frac{1}{2} \sum_{I}^{3 N} \dot{q}_{I}^{2}+\frac{1}{2} \sum_{I}^{3 N} \sum_{J}^{3 N} K_{I J} q_{I} q_{L L}$
If we diagonalize $\mathrm{K}_{\mathrm{IJ}} \quad E=\frac{1}{2} \sum_{I}^{3 N-5 / 6} \dot{Q}_{I}^{2}+\frac{1}{2} \sum_{I}^{3 N-5 / 6} \lambda_{I} Q_{I}^{2}$
Motion described by 3N-6(5 for linear molecule) NORMAL MODES

$$
Q_{I}=\sum_{J=1}^{3 N} L_{I J} q_{J} \quad I=1,3 N-5 / 6
$$

5/6 Modes that are ignored have zero $\lambda_{\mathrm{I}}$ TRANSLATION/ROTATI_ON

