OUTLINE OF "PRE-CALCULUS (基礎數學)" IN THE WINTER SEMESTER 2016 (NTU)

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1. TARGET PARTICIPANTS

Students (outside of the Department of Mathematics) who are attending (or intend to attend) the course "Calculus" and would like to seek help in reviewing and strengthening the background needed for the course.

2. OBJECTIVES

(1) To prepare students for the course "Calculus" (or to assist them if they are taking the course).
(2) To familiarise students with the calculations involved in this course.
(3) To encourage students to relate topics appearing in this course with their own fields of study.

3. CONTENTS OF THE COURSE

Topics presented below will be discussed more or less in the following order, although some of them may be reordered in order to fit in the pace of teaching of the course "Calculus".

3.1. **First semester.**

Unit 1 : Numbers and Functions
    (a) An overview of the *real (and complex) number system*;
    (b) a quick revision of *exponents*, *radicals*, *polynomials*, *rational functions* and trigonometric functions;
    (c) the concept of *functions* and their graphs: practical examples;
    (d) formalities of functions: sets, relations, functions, domains, ranges, injectivity, surjectivity, monotonicity and inverses.

    Irrational numbers are introduced via the existence of non-commensurable lengths. The proof of irrationality of $\sqrt{2}$ may be given to illustrate the use of contra-positivity in mathematical proofs, and to recall the factorisation of integers.

Unit 2 : Exponential and Logarithmic functions
    (a) *Exponential functions* defined on $\mathbb{Q}$ and on $\mathbb{R}$, and their graphs;
    (b) properties of exponential functions;
    (c) compound interest and the number e;
    (d) *Logarithms* defined on $\mathbb{R}_{>0}$ and their graphs;
    (e) properties of logarithms and changing bases;

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(f) solving equations and inequalities involving exponential and logarithmic functions.

Exponential functions are discussed before polynomial functions in order to synchronise with the discussion in the course "Calculus".

Aiming at familiarising students with the algebraic operations of the functions.

Unit 3 : Polynomials and Rational functions
(a) *Linear and Quadratic functions* revisit: slope, extrema, roots and their graphs;
(b) factorisation and division algorithm;
(c) roots of polynomials and the Fundamental Theorem of Algebra;
(d) difference quotients of polynomials and primitive version of derivatives;
(e) graphs of polynomials and rational functions.

Focus should be put on sketching the graphs of polynomials and rational functions, especially with the help of the extra information from their derivatives.

Unit 4 : Trigonometric functions
(a) Definitions from the unit circle and the Pythagorean identities;
(b) parity, periodicity and graphs of trigonometric functions;
(c) area of triangles, *sine law* and *cosine law*;
(d) exponential functions over $\mathbb{C}$ and Euler's formula;
(e) De Moivre's formula vs addition and subtraction formulas of sines and cosines;
(f) double-angle, half-angle and product-to-sum formulas;
(g) applications to geometric problems.

Exponential functions over $\mathbb{C}$ are introduced without justifying their well-definedness. Presenting addition and subtraction formulas via de Moivre's formula provides an easier proof of the formulas. Hopefully this would ease the difficulties for students to memorise and to understand the formulas.

Unit 5 : Conic sections express
(a) Equations of conic sections in regular form and their graphs.

Conic sections are discussed at this point in order to facilitate the discussion on conic sections in polar coordinates in the course "Calculus".

Emphasis is put on students' ability to read off the graph of a conic from its formula.

3.2. **Second semester.**

Unit 6 : Sequences and Series
(a) *Sequences, series, partial sums* and their examples;
(b) *Arithmetic sequences and series*;
(c) *Geometric sequences and series*;
(d) *Mathematical Induction* and formulas of sum of powers;
(e) the Binomial Theorem and formulas relating the binomial coefficients.

Sequences and series can be introduced as the discrete version of functions and integrals.

Students should be familiarised with the operations under the summation sign.

Unit 7 : Vectors in 2D and 3D
   (a) Definition, addition and scalar multiplication of vectors;
   (b) dot product and orthogonal projection;
   (c) cross product and volume of parallelepiped;
   (d) equations of lines and planes.

   This is to facilitate the discussions on vector functions, space curves, directional derivatives as well as line and surface integrals in the course "Calculus".

Unit 8 : Systems of linear equations and Matrices
   (a) Methods of solving systems of linear equations;
   (b) systems of linear equations in matrix form;
   (c) elementary row operations and Gaussian elimination;
   (d) classification of solutions of systems of linear equations;
   (e) algebraic operations on matrices;
   (f) determinants and inverses of square matrices;
   (g) Cramer's rule;
   (h) matrices as rotations and translations on vectors.

   Students should be familiarised with the elementary matrix operations as well as the calculation of determinants.

Unit 9 : Conic sections revisit
   (a) General equation of a conic in matrix form;
   (b) rotation of axes and discriminant.

   This is to illustrate the use of matrices to represent linear change of coordinates has the advantage in simplifying calculations.