ECON5226: Introductory Stochastic Calculus with Applications to Economics and Finance (Updated August 5th, 2024)

TUE 678 — Fall 2024

Instructor Information

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Office Hours: After each class or by appointment

Class Information

Dates: September 2 – December 13

Time: Tuesday 13:20 to 16:20

Classroom: 社科407

Course Description

This course introduces fundamental mathematical tools for stochastic calculus in an accessible manner, emphasizing their practical applications in formal modeling within economics and finance. Examples of such models include the Black-Scholes option pricing model, consumption-saving-portfolio choice problems, and basic dynamic general equilibrium models used in macroeconomics and asset pricing.

The mathematical techniques covered in this course extend far beyond these examples, providing a foundation applicable to a wide range of contexts. It serves as a preparatory course for more advanced techniques utilized in the financial industry and in graduate-level courses on macroeconomics and asset pricing.

Prerequisites: undergraduate-level calculus, linear algebra, calculus-based probability (or statistics but I do not require the knowledge of hypothesis testing) and very basic topology (set operations, elementary concepts such as open sets).

Course Objectives

After this course, you should be able to. . .

- To compute basic option prices analytically.
- To characterize and solve dynamic optimal control problems.
- To derive the basic asset pricing equations and understand the underlying economics.
- To understand intuitions of measure theoretic probability theory.
- To apply stochastic calculus to the problems relevant in economics and finance.
- To comprehend the key economic intuitions and mathematical framework which are broadly applicable beyond the applications covered in this course.
- To understand many prerequisites for more advanced modeling on the frontier of macroeconomics and asset pricing.

Textbook

Textbook: There are no mandatory readings for this course. Lecture notes will be comprehensive and self-contained. However, students are highly encouraged to refer to the following textbook, which serves as an excellent introduction to the subject matter. The textbook provides numerous simple examples that illustrate abstract mathematical concepts in stochastic calculus.

Informal Introduction to Stochastic Calculus with Applications, by Calin (2022).

Quick, concise, but solid introduction to measure-theoretic probability theory.
Probability with Martingales, by Williams (1999).

Several chapters and appendices deeply discuss mathematical details of the applications beyond the aim of this course.

Dynamic Asset Pricing Theory, by Duffie (2001).

Class Attendance and Participation

Attending every lecture is highly recommended since homework and exams are closedly connected to the contents I discuss in the lecture.

Homework

Students are required to draft their answers independently and submit problem sets approximately every two weeks, with each set due within the following two weeks. I will return the problem sets after verifying that adequate effort has been put into the work, awarding full marks if this criterion is met, but without providing detailed corrections. Answer keys will be posted after the submission deadlines, allowing students to evaluate their own answers. I recommend keeping a copy of your answers, as I cannot guarantee timely return of the checked homework. Late submissions will not be accepted without a reasonable excuse.

Many questions on the exams will be similar to those in the problem sets, so completing these assignments is an effective way to prepare for the exams. While students must draft their answers independently, I highly recommend collaborating with other students during the problem-solving process.

Exams

There are a midterm and a final exam. You are expected to be present, seated, and ready to take the exam before the exam begins. The exams are schedule for the week of 8 and 16, respectively. You are not permitted to use any outside materials, resources, or electronic devices (including but not limited to mobile phones, smartwatches, etc., but not including a calculator) on the exams. Any violation of this policy is a violation of the university's Academic Integrity Policy.

There will be no make-up exam, even in the case of an emergency. There will be no make-up exams offered. In the event of a missed midterm exam due to a medical emergency, students must provide

a doctor's note within one week of the exam. If such documentation is provided, I will appropriately adjust the weighting of their scores from the final exam.

For anticipated medical conditions, students are required to notify me via email in advance. Attendance at the final exam is mandatory.

The final exam will cover the entire course content, with a greater focus on the latter half. All material provided in the course is examinable, unless explicitly specified otherwise.

If a student believes their exam was graded incorrectly, they may request a regrade within one week of receiving the graded exam. To do so, they must submit a written note explaining the perceived grading error. I will then regrade the entire exam, which may result in an adjustment of the score, either upward or downward. Please note that there are no additional discretionary methods available for improving scores.

Grading

The course grade is determined by the following components:

Problem sets	20%
Midterm exam	30%
Final exam	50%

Grade Scale

Final grades will be assigned according to the following scale:

A+	90 – 100	B-	65 – 69
A	85 – 89	C+	60 – 64
A-	80 – 84	C	55 – 59
B+	75 – 79	C-	50 – 54
B	70 – 74	F	0 – 49

Email Policy

I typically answer emails quickly, almost always within a day. Please start the subject headline of all your emails to me with ECON5226. Otherwise, I might miss your emails. Also, before sending any emails, please read the syllabus. This may save you a lot of time.

Tentative Schedule

The following is a *tentative* schedule for the course.

Week	Sections	Week	Sections
1 (9/3)	Introduction and Probability Theory	9 (10/29)	Stochastic Integration
2 (9/10)	Probability Theory	10 (11/5)	Stochastic Differentiation
3 (9/17)	No class (holiday)	11 (11/12)	Stochastic Differentiation
4 (9/24)	Probability Theory	12 (11/19)	Arbitrage Pricing
5 (10/1)	The Useful Stochastic Processes	13 (11/26)	Applications: Black-Scholes option pricing
6 (10/8)	Properties of Stochastic Processes	14 (12/3)	Applications: Dynamic Portfolio Choices
7 (10/15)	Properties of Stochastic Processes	15 (12/10)	Applications: General Equilibrium Asset Pricing
8 (10/22)	Midterm Exam	16 (12/17)	Final Exam

Calin, Ovidiu. 2022. *An Informal Introduction to Stochastic Calculus with Applications* World Scientific (2nd edition).

Duffie, Darrel. 2001. *Dynamic Asset Pricing Theory* Princeton University Press.

Williams, David. 1999. *Probability with Martingales* Cambridge University Press.