ECON5226: Introductory Stochastic Calculus with Applications to Economics and Finance

WED 789 — Fall 2025

Instructor Information

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TA Information

Name: TBA

Email: TBA

Class Information

Dates: September 3 to December 17

Time: Wednesday 14:20 to 17:20

Classroom: COSS 507

Course Description

This course introduces fundamental mathematical tools for stochastic calculus in an accessible manner, using the asset pricing theory as the primary examples and illustrating their practical applications in formal modeling within economics and finance. Examples of such models include the arbitrage pricing theory, the Black-Scholes option pricing model, consumption-saving-portfolio

choice problems, learning and filtering, and basic dynamic general equilibrium models widely used in macroeconomics and asset pricing.

The abstract concepts in the probability theory and stochastic calculus will be discussed in the concrete examples from the asset pricing theory, which helps fill the gap between abstract mathematics and economics. Nevertheless, the mathematical techniques covered in this course extend far beyond these examples, providing a foundation applicable to a wide range of contexts. It serves as a preparatory course for more advanced techniques utilized in the financial industry and in graduate-level courses on macroeconomics and asset pricing. For example, the knowledge learned in this course can be used in the subsequent graduate-level course about continuous-time models in macroeconomics and asset pricing that I will teach in Spring semester (ECON7233).

I will sometimes skip or just briefly sketch detailed mathematical proofs if they do not contain substantial economic substances. However, several references are provided for proofs so that interested students can learn more about detailed mathematics, though I will not require to do so for problem sets or exams.

We begin the course by studying the basic asset pricing theory to illustrate the abstract mathematical concepts in probability theory with the minimum mathematical complexity in Topic 1. Then we learn about the fundamental mathematics of probability theory (Topic 2) and continuous-time stochastic calculus (Topic 3). Equipped with this knowledge, we revisit the asset pricing theory in a continuous-time environment in Topic 4. Then we apply those knowledge to many applications in economics and finance in the remaining topics (see the class schedule below).

Prerequisites: undergraduate-level calculus, linear algebra, calculus-based probability (or statistics but I do not require the knowledge of hypothesis testing). Basic knowledge of real analysis is helpful to deeply understand the class discussions but not required to solve problem sets and exams.

Course Objectives

After this course, you should be able to...

- To understand abstract mathematical concepts in probability theory and stochastic calculus in the contexts of concrete applications of economic theories.

- To understand the fundamental concepts in the asset pricing theory.
- To apply the asset pricing theory to the basic derivative pricing.
- To characterize and solve dynamic optimal control problems.
- To understand and solve dynamic optimization problems in continuous time, learning and filtering, and basic general-equilibrium asset pricing.
- To understand many prerequisites for more advanced modeling on the frontier of macroeconomics and asset pricing in continuous time.

Textbook

Textbook: There are no mandatory readings for this course. Lecture slides and supplementary lecture notes will be comprehensive and self-contained. However, students are highly encouraged to refer to the following textbooks to complement the information in slides. I will provide relevant references throughout the course.

- The following textbook provides numerous simple examples that illustrate abstract mathematical concepts in stochastic calculus in simple manners:

  Informal Introduction to Stochastic Calculus with Applications, by Calin (2022).

- You can find most of the mathematical foundations taught in this course here:

  Stochastic Differential Equations, by Øksendal (2007).

- Quick, concise, but solid introduction to measure-theoretic probability theory:

  Probability with Martingales, by Williams (1999).

- Detailed textbook of the probability theory with economic applications written by one of top decision-theory economists:

  Probability Theory and Measure with Economic Applications, by Ok

- Several chapters and appendices deeply discuss mathematical details of the applications beyond the aim of this course.

  Dynamic Asset Pricing Theory, by Duffie (2001).

Class Attendance and Participation

Attending every lecture is highly recommended since homework and exams are closely connected to the contents I discuss in the lecture.

Homework

6 problem sets will be assigned in total. Each problem set is designed for each topic listed in the class schedule below. Students are required to draft their answers independently and submit problem sets to the NTUCOOL's course page within approximately two weeks after they are assigned. I will grade problem sets after verifying that adequate effort has been put into the work, awarding full marks if this criterion is met, but without providing detailed corrections. Answer keys will be posted after the submission deadlines, allowing students to evaluate their own answers. Late submissions will not be accepted without a reasonable excuse.

Many questions on the exams will be similar to those in the problem sets, so completing these assignments is an effective way to prepare for the exams. While students must draft their answers independently, I highly recommend collaborating with other students during the problem-solving process.

Exams

There will be a midterm and a final exam. You are expected to be present, seated, and ready to start the exam before it begins. The midterm is scheduled for the week 8 and covers the questions in Topic 1, 2, 3 in the class schedule below. The final exam is scheduled for the week of the 16th and covers all the topics with a greater focus on the latter half (Topic 4, 5, 6). All materials on slides are examinable unless notified.

For the midterm exam, you are allowed to bring one A4-size cheat sheet. For the final exam, you may bring two A4-size cheat sheets. No other outside materials, resources, or electronic devices (including mobile phones, smartwatches, etc., but not including calculators) are permitted during the exams. Any violation of these policies will be considered a breach of the university's Academic Integrity Policy.

There will be no make-up exam, even in the case of an emergency. There will be no make-up exams

offered. In the event of a missed midterm exam due to a medical emergency, students must provide a doctor's note within one week of the exam. If such documentation is provided, I will appropriately adjust the weighting of their scores from the final exam.

For anticipated medical conditions, students are required to notify me via email in advance. Attendance at the final exam is mandatory.

If a student believes their exam was graded incorrectly, they may request a regrade within one week of receiving the graded exam. To do so, they must submit a written note explaining the perceived grading error. I will then regrade the entire exam, which may result in an adjustment of the score, either upward or downward. Please note that there are no additional discretionary methods available for improving scores.

Grading

The course grade is determined by the following components:

Problem sets	20%
Midterm exam	30%
Final exam	50%

Grade Scale

Final grades will be assigned according to the following scale subject to change:

A+	90 – 100	B-	65 – 69
A	85 – 89	C+	60 – 64
A-	80 – 84	C	55 – 59
B+	75 – 79	C-	50 – 54
B	70 – 74	F	0 – 49

Email Policy

I typically answer emails quickly, almost always within a day. Please start the subject headline of all your emails to me with ECON5226. Otherwise, I might miss your emails. Also, before sending any emails, please read the syllabus. This may save you a lot of time.

Tentative Schedule

The following is a *tentative* schedule for the course.

Week	Sections	Notes
1 (9/3)	Topic 1: Foundations of Asset Pricing Theory	Problem Set 1 posted
2 (9/10)	Topic 1: Foundations of Asset Pricing Theory	
3 (9/17)	Topic 2: Probability Theory	Problem Set 1 due and Problem Set 2 posted
4 (9/24)	Topic 2: Probability Theory	
5 (10/1)	Topic 3: Continuous-Time Stochastic Calculus	Problem Set 2 due and Problem Set 3 posted
6 (10/8)	Topic 3: Continuous-Time Stochastic Calculus	
7 (10/15)	Topic 4: Arbitrage Pricing in Continuous Time	Problem Set 3 due and Problem Set 4 posted
8 (10/22)	Midterm Exam	
9 (10/29)	Topic 4: Arbitrage Pricing in Continuous Time	
10 (11/5)	Topic 5: Asset Pricing Applications	Problem Set 4 due and Problem Set 5 posted
11 (11/12)	Topic 5: Asset Pricing Applications	
12 (11/19)	Topic 5: Asset Pricing Applications	
13 (11/26)	Topic 6: Learning and Filtering	Problem Set 5 due and Problem Set 6 posted
14 (12/3)	Topic 6: Learning and Filtering	
15 (12/10)	Buffer & Office Hours	Problem Set 6 due
16 (12/17)	Final Exam	

Calin, Ovidiu. 2022. *An Informal Introduction to Stochastic Calculus with Applications* World Scientific (2nd edition).

Duffie, Darrel. 2001. *Dynamic Asset Pricing Theory* Princeton University Press.

Ok, Efe. ???? *Probability Theory and Measure with Economic Applications*. URL https://sites.google.com/a/nyu.edu/efeok/books#h.ahhx5eckso96.

Williams, David. 1999. *Probability with Martingales* Cambridge University Press.

Øksendal, Bernt. 2007. *Stochastic Differential Equations* Springer (6th edition).