Course Description for "Partial Differential Equations"

Contents:

This is a "second course" about an introduction to PDE. The students taking this course are assumed to be familiar with the contents of Chapter 1 to Chapter 11 of the text "Partial Differential Equations, an Introduction" by W.A. Strauss. The first-order PDE, second-order linear elliptic equations, second-order linear parabolic equations and second-order linear hyperbolic equations are the four main subjects in PDE to be more thoroughly discussed in this course. And if time permits, we will discuss the Variational Methods, or Reaction-diffusion Equations, or Non-linear Wave Equations depending on our progress. The textbook "Partial Differential Equations" by L.C. Evans is our main textbook. We will not cover every chapter in this text. However, we may supplement some additional materials to the topics we discuss.

There will be a series of Homeworks, and the students are required to work out these Homeworks. There are a Midterm and Final in each semester.

A. The method of charateristics for first-order PDE. Viscosity solutions for Hamilton-Jacobi Equations. Weak solutions for scalar conservation laws satifying entropy conditions.

B. Reviews on the Laplacian. Maximum principles for second order linear elliptic operators. Weak solutions via the Lax-Milgram theorem. Schauder estimates. Regularity via the Sobolev embedding theorems. The Harnack inequality. The eigenvalue problems and the mini-max principles.

C. Reviews on the heat equations. Maximum principles for second-order parabolic equations. The weak (or mild) solutions via a Galerkin approximations. Parabolic Schauder estimates and regularity of weak solutions. The Harnack inequality. The semi-group generated by a second-order elliptic operator.

D. Reviews on the wave equations. Weak solutions to a second-order hyperbolic equations via Galerkin approximations. Energy estimates and regularity of the weak solutions. Domain of dependence and range of influence. Linear hyperbolic systems.

Course prerequisite:

Linear Algebra, Advanced Calculus (or Real Analysis), PDE course in junior year.

Reference materials:

Main Text: Evans, L.C., Partial Differential Equations, AMS Graduate Studies in Mathematics, Vol. 19.
References:

(a) Strauss, W.A., Partial Differential Equations, an Introduction.

(b) John, F., Partial Differential Equations.

Grading schemes:

Homeworks (30 percent), Midterm (35 percent), Final (35 percent).

Course Goal:

The students are expected to learn the more theoretical aspects of the elementary PDE theory. We hope that these knowledge will help the students to get more easily into the field of modern PDE (especially the non-linear ones) theory.