A Description of the Course： Probability Theory and Real Analysis

The modern theory of finance has been rapidly developing, since first came the 1952 publication of "Portfolio Selection" by H. Markowitz, then the famous formula for the value of European call option derived in 1973 by Black and Scholes. Modern probability theory has played crucial role in the further development.

The original aim of this course is to give a comprehensive introduction to the modern probability theory for the students who are interested in the mathematical theory of finance. The course is arranged in two semesters. The first semester covers basic probability theory. The second semester covers stochastic calculus and its applications. It is fair to say that the mathematical theory of modern finance is built on the theory of stochastic calculus. As an advanced topic in probability theory, stochastic calculus has been developed into a sophisticated mathematical theory, partly because of its broad applications, especially for its connection to other branches of mathematics such as the theory of partial differential equations, differential geometry, etc. The measure theory has been heavily used in the development. Therefore, a good understanding of stochastic calculus requires a solid knowledge on measure theoretical approach probability theory. For this reason, one of important part of the course in the first semester is to introduce the measure theoretical approach probability theory. The measure theory will be used in defining the expectation of a random variable, the conditional expectation, etc.

The materials are chosen based on the following considerations. First, the course is designed for Ph.D students, and advance master students. To help students to understand and appreciate the probability theory and the theory of stochastic calculus, and eventually they can use in their research, we think it will be helpful for them to know the analysis behind the theorems. Secondly, we realize the students in the class have diversified background of training in mathematics, and most of them the training are weak. This lack of mathematics training is a great barrier for the students to understand a deep mathematic analysis as well as an advanced mathematics theory such as stochastic calculus. Therefore, the lectures will be organized so that they will gradually help the students to build a sense of doing mathematics analysis. For this purpose, the theory and important concepts will be introduced using discrete objects, then theory of limit will be introduced, and finally we can discuss more abstract theory. Main part of the lectures is the discussion of concepts and the mathematical analysis for theorems.

There will be weekly homework. Most of the problems in the homeworks are chosen to help the students to understand the important concepts and theorems. We think this is a very important part of learning process. We carefully select book(s) for the course. The students are encouraged to read the book as much as they could. Reading will be also important when they later search a concept or a result in a book or an article for their research.

Main Text Book

Jean Jacod and Philip Protter, Probability Essentials, 2003, 2nd edition, Springer-Verlag

Additional Reading

Introductory Level

Sheldon Ross, A first Course in Probability, 1994, 4th Edition.

W. Feller, An introduction to probability theory and its applications, 1967, Vol. I, 3rd edition, Wiley, New York.

Intermediate Level

A. F. Karr, Probability, 1993, Springer-Verlag

Advanced Level

R. Durrett , Probability: Theory and Examples, 1996, Second Edition. Duxbury Press.

K. L. Chung, A Course in Probability Theory, 1974, Second Edition. Academic Press, New York.

L. Breiman, Probability,1968, Addison-Wesley

Mathematical Analysis

S. K. Berberian, A First Course in Real Analysis, 1994, Springer-Verlag

Suggestion:

We shall mainly follow the book by Jacod and Protter. However, it will be very helpful for the students to do the following before the class starts. Read parts of Ross' book (as much as you could). It has good examples and can give you some good idea about some important concepts and possible applications of probability theory. Read first few chapters of Berberian's book. This gives you some idea how to do mathematical analysis (or mathematical reasoning). We will need this more when we start to introduce measure theoretical approach of probability theory. It is even also helpful when we discuss concepts and theorems for discrete probability space. You can also use other undergraduate level textbooks which have similar content. There are many such books available in the library.

The following is the content of the lectures.

Lecture 1: Introduction
Lecture 2. Axioms of Probability
Lecture 3. Conditional Probability and Independence