Course Description for "Stability Anaysis of Travelling Waves"

Contents:

The travelling waves are a common natural phenomenon in a lot of wave propagation problems, such as the water waves, the EM waves, or the plane waves etc.. Typically, such waves are self-similar solutions of a system which is invariant under the Euclidean group actions. Even a diffusion or dispersive process coupled with some non-linear effects can be balanced to generate travelling waves. Prominent examples are the spread of the dominant species in an ecological system, the advance of an advantageous gene in genetic heritage problems, the appearence of the spiral waves in cardiac arrhythmias, the detonations or deflagrations in a combustible media, or the solitons appearing in so many non-linear dispersive wave theory, etc.. Those travelling waves observed in nature are the "stable" ones. How to detect stability of a travelling wave is the main content of this course.

A. Three methods to show the existence and uniqueness of travelling waves. (a) The homotopy method using the Leray-Schauder degrees. (b) The topological method using the Conley indices. (c) The method of geometric singular perturbations.

B. Local stability theory. The spectra of linearized operators. The Palmer dichotomy theorems. The Evans functions. The stable bundles.

C. Global stability theory. Scalar equations. Monotonic systems. Lotka-Volterra systems. Combustion waves.

D. Generic instability of tranverse perturbations of a planar travelling wave in 2-D or 3-D. Breaking the Euclidean symmetry. Spirals and Scrolls.

E. The interface equations derived from the singular limits of spiral or scroll waves. Motions of the interfaces by the mean-curvature flow. The viscosity solutions.

Course prerequisite:

Linear Algebra, Complex Variables, Real Analysis, PDE.

Reference materials:

(a) Lecture notes given by the Lecturer in the Applied Mathematice Department of Providence University, 2007.

(b) Volpert, A.I., Volpert, V.A. and Volpert, V.A., Travelling Wave Solutions of Parabolic Systems, AMS Translation Monographs, Vol. 140, 1994.

(c) Some original papers given along the progress of the course.

Grading schemes:

The grades are based on the Homework Assignments. There will be two problems for every two weeks.

Course Goal:

The Lecturer hopes that the subject of this course can provide a linkage between mathematics and its applications to model the natural phenomenon of the travelling waves.