林约韩 选修课

Course Description for "Mathematical Theory of Fluid Mechanics"

Contents:

We will spend one and half semesters on the incompressible fluid flow defined by the Navier-Stokes equations (and the Euler equations), and then touch on the subject of compressible gas flow. The students are assumed to have a solid understanding of real analysis (on the level of the text by Wheeden and Zygmund) and PDE theory (on the level of the text by Evans). We list the following main contents.

A. Helmholtz decompositions and the semigroup generated by the Stokes operator.

B. Leray's global weak solutions, and local strong solutions. The roles of weak energy inequality.

C. Uniqueness and regularity consideration of weak solutions. The Cafferelli-Kohn- Nirenberg estimates.

D. Vorticity and singularity formations in Euler equations. Vortex systems.

E. Kolomogroff theory of homogeneous turbulence. Global compact attractors of Navier-Stokes flow. Statistical interpretations of turbulent flows.

F. The thermodynamical aspects of compressible gas flow. Shock and rarefaction waves in 1-d. Shock formations in 1-d. The entropy conditions and the uniqueness problems of compressible gas flow in 1-d.

Course prerequisite:

Real Analysis, ODE, PDE.

Reference materials:

(a) Foias, Manley, Rosa and Teman, Navier-Stokes Equations and Turbulence.

(b) Constantine and Foias, Navier-Stokes equations.

(c) Lions, Mathematical Topics in Fluid Mechanics, Vol 1 and 2.

(d) Some original papers along the progress of the course.

Grading schemes:

There will be one take-home problems in each semester.

Others:

Course Goal:

Hopefully, this course will achieve an in-depth understanding of the Navier-Stokes flow.